OCR
Oxford Cambridge and RSA

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Question Set 3

4 **(a)** Find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x$. **[4]**

$3x^2 - 12x + 9 = 0 \quad x^2 - 4x + 3 \quad x = 1 \quad x = 3 \quad (1,3)$
$y = 4 \quad y = 0 \quad (4,0)$

(b) The equation $x^3 - 6x^2 + 9x + k = 0$ has exactly one real root.

Using your answers from part **(a)** or otherwise, find the range of possible values of k. **[2]**

$k > 0$ as shifts min point up $\therefore$ only goes through X axis once

5 **(a)** Prove that the following statement is **not** true.

m is an odd number greater than $1 \Rightarrow m^2 + 4$ is prime. **[1]**

$m = 9 \quad 9^2 + 4 = 85 \rightarrow$ not prime

(b) By considering separately the case when n is odd and the case when n is even, prove that the following statement **is** true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4. **[4]**

even $\quad (2n)^2 + 1$
$= 4n^2 + 1 \therefore$ not multiple of 4

odd $\quad (2n+1)^2 + 1 = 4n^2 + 4n + 2$
$= 4(n^2 + n) + 2 \therefore$ not multiples of 4

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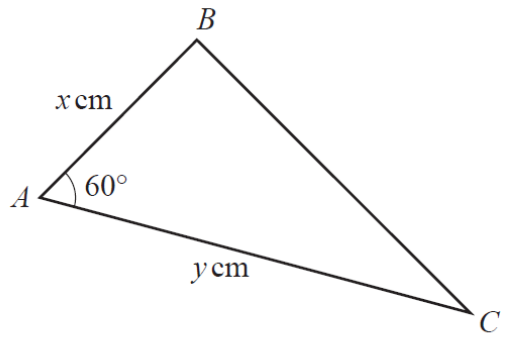

The diagram shows triangle ABC, with $AB = x$ cm, $AC = y$ cm and angle $BAC = 60°$. It is given that the area of the triangle is $(x+y)\sqrt{3}$ cm^2.

(a) Show that $4x + 4y = xy$. **[2]**

$0.5 \times 4 \times \sin 60 = (x+y)\sqrt{3}$

$0.25 \times 4 \times \sqrt{3} = (x+y)\sqrt{3}$ $0.25xy = x + y$ $xy = 4(x+y)$

When the vertices of the triangle are placed on the circumference of a circle, AC is a diameter of the circle.

(b) Determine the value of x and the value of y. **[4]**

Since AC is a diameter, $ABC = 90^0$ $\therefore \sin 30 = \frac{x}{y}$ $y = 2x$

$4x + 4y = xy$ $4x + 8x = 2x^2$ $12 = 2x$ $x = 6$ $y = 12$

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(a) Write down an expression for the gradient of the curve $y = e^{kx}$. **[1]**

ke^{kx}

(b) The line L is a tangent to the curve $y = e^{\frac{1}{2}x}$ at the point where $x = 2$. $\rightarrow y = e$

gradient $= \frac{1}{2}e$

Show that L passes through the point (0, 0). **[4]**

$y = \frac{1}{2}xe + C$ $e = e + C$ $\therefore C = 0$ $y = \frac{1}{2}xe$ $\therefore$ goes through (0,0)

(c) Determine the coordinates of the point of intersection of the curves $y = 3e^{x}$ and $y = 1 - 2e^{\frac{1}{2}x}$. **[6]**

$3e^{x} = 1 - 2e^{\frac{x}{2}}$ let $e^{x} = m^{2}$ $3m^{2} = 1 - 2m$ $3m^{2} + 2m - 1 = 0$ $(3m - 1)(m + 1)$ $m = \frac{1}{3}$ $m = -1$

as $m^{2} = e^{x}$ $m > 0$

$\therefore m = \frac{1}{3}$

$m^{2} = \frac{1}{9} = e^{x}$

$e^{x} = \frac{1}{9}$

$x = \ln \frac{1}{9}$

Total Marks for Question Set 3: 50