

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Question Set 3

- 4 (a) Find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x$. [4]

$$3x^2 - 12x + 9 = 0 \quad x^2 - 4x + 3 \quad \begin{matrix} x=1 & x=3 & (1,3) \\ y=4 & y=0 & (4,0) \end{matrix}$$

- (b) The equation $x^3 - 6x^2 + 9x + k = 0$ has exactly one real root.

Using your answers from part (a) or otherwise, find the range of possible values of k . [2]

$k > 0$ as shifts min point up. \therefore only goes through x-axis once

- 5 (a) Prove that the following statement is **not** true.

m is an odd number greater than 1 $\Rightarrow m^2 + 4$ is prime. [1]

$$m=9 \quad 9^2 + 4 = 85 \Rightarrow \text{not prime}$$

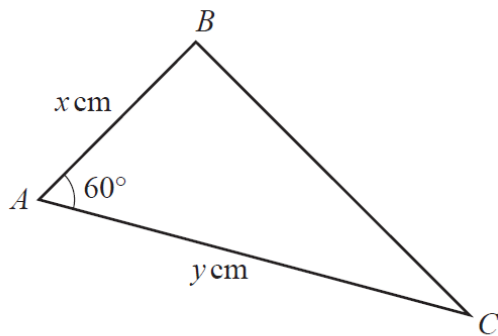
- (b) By considering separately the case when n is odd and the case when n is even, prove that the following statement is true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4. [4]

even $(2n)^2 + 1$
 $= 4n^2 + 1 \therefore$ not multiple of 4

odd $(2n+1)^2 + 1 = 4n^2 + 4n + 2$
 $= 4(n^2 + n) + 2 \therefore$ not multiple of 4

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The diagram shows triangle ABC , with $AB = x$ cm, $AC = y$ cm and angle $BAC = 60^\circ$. It is given that the area of the triangle is $(x+y)\sqrt{3}$ cm².

(a) Show that $4x + 4y = xy$. [2]

$$0.5xy \times \sin 60 = (x+y)\sqrt{3}$$

$$0.25xy\sqrt{3} = (x+y)\sqrt{3} \quad 0.25xy = x+y \quad xy = 4(x+y)$$

When the vertices of the triangle are placed on the circumference of a circle, AC is a diameter of the circle.

(b) Determine the value of x and the value of y . [4]

$$\text{Since } AC \text{ is a diameter, } \angle ABC = 90^\circ \therefore \sin 30 = \frac{x}{y} \quad y = 2x$$

$$4x + 4y = xy \quad 4x + 8x = 2x^2 \quad 12 = 2x \quad x = 6 \quad y = 12$$

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(a) Write down an expression for the gradient of the curve $y = e^{kx}$. [1]

$$ke^{kx}$$

(b) The line L is a tangent to the curve $y = e^{\frac{1}{2}x}$ at the point where $x = 2$. $\rightarrow y = e$
 Show that L passes through the point $(0, 0)$. $\text{gradient} = \frac{1}{2}e$ [4]

$$y = \frac{1}{2}e + c \quad e = e + c \therefore c = 0 \quad y = \frac{1}{2}e \therefore \text{passes through } (0, 0)$$

(c) Determine the coordinates of the point of intersection of the curves $y = 3e^x$ and $y = 1 - 2e^{\frac{1}{2}x}$.

[6]

$$\begin{aligned} 3e^x &= 1 - 2e^{\frac{x}{2}} & 3m^2 &= 1 - 2m & (3m-1)(m+1) & \text{as } m^2 = e^x \\ \text{let } e^x &= m^2 & 3m^2 + 2m - 1 &= 0 & m = \frac{1}{3} \quad m = -1 & m > 0 \\ & & & & \therefore m &= \frac{1}{3} \\ & & & & m^2 &= \frac{1}{9} = e^x \\ & & & & e^x &= \frac{1}{9} \\ & & & & x &= \ln \frac{1}{9} \end{aligned}$$

Total Marks for Question Set 3: 50

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